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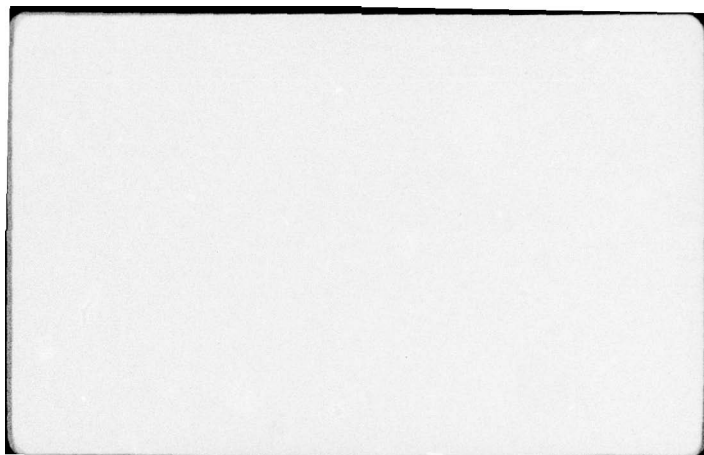   RECENT PROGRESS IN THE DYNAMIC PLASTIC BEHAVIOR OF STRUCTURES.(U)
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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF OCEAN ENGINEERING

CAMBRIDGE, MASS. 02139

RECENT PROGRESS IN THE DYNAMIC PLASTIC

BEHAVIOR OF STRUCTURES.

by

Norman/Jones
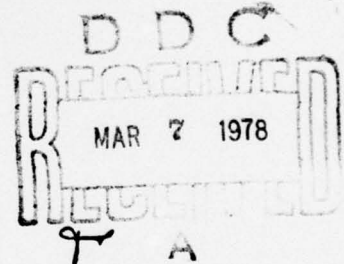
Report Number 78-1

Jan 1978

Distribution Unlimited

This report will be published in The Shock and Vibration Digest.

Index

Abstract

This article attempts to review the literature on the dynamic
plastic response of structures published during the last three years,
since the last survey by the author was published in the Shock and Vibra-
tion Digest (1). The review focuses largely on the behavior of simple
structural components such as beams, plates and shells subjected to
dynamic loads which produce extensive plastic flow of the material.
In particular, recent work on the behavior of ideal fibre-reinforced
beams, higher modal response of beams, the influence of transverse shear
and rotatory inertia, approximate methods of analysis, rapidly heated
structures, fluid-structure interaction and dynamic plastic buckling
are discussed in detail. These topics are followed by a discussion of
a few recent numerical studies on the dynamic plastic response of struc-
tures, and a brief survey of some recent experimental and theoretical
investigations into the collision protection of vehicles.

1. Introduction

It is the object of this article to survey the recent literature published on the inelastic response of structural members (beams, plates and shells) subjected to dynamic loads or suddenly applied displacements, which are responsible for permanent displacements or damage of structures. The results of studies in this particular area may be used for a wide variety of applications in a number of different fields. For example, the conclusions of such studies are guiding the development of rational design procedures to avoid the destructive action of earthquakes on buildings, being employed to improve occupant safety during collisions of aircraft, automobiles, buses and trains, designing the collision protection of ships and marine vehicles containing hazardous cargoes, estimating slamming and bow wave damage of ships and marine vehicles, designing nuclear reactor tubes to withstand violent transient pressure pulses, designing buildings to withstand internal gaseous explosions, and designing energy absorbing systems for various applications. In order to avoid unnecessary repetition, a reader who requires additional background for this general field may consult section 1 of Reference (1), which is the last* review of this subject to be published in the Shock and Vibration Digest.

Krajcinovic (2) surveyed the exact theoretical solutions available on the dynamic inelastic behavior of various rigid perfectly plastic structures which undergo infinitesimal displacements. The review in

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*The last review at the time of writing this article. Reference (128) has since been published.

Reference (1) was made purposely complementary to Reference (2) in the
sense that it focussed largely on the non-linear effects of finite-
displacements, or geometry changes, and material strain rate sensi-
tivity, both of which are important in many practical situations.   In
addition to these two articles, Baker (3) published a survey in the
Shock and Vibration Digest on approximate techniques for estimating the
plastic deformation of structures acted on by impulsive loads.
Furthermore, Rawlings (4) reviewed the work contained in over 120
references on a wide range of metal structures subjected to dynamic over-
loads and discussed many applications, particularly concerning auto-
mobile safety.

It emerges clearly from the previous comments that the literature
published on the dynamic plastic response of structures has been ade-
quately reviewed up to 1974, when References (1, 3, 4) were prepared.
The task of this article is therefore to attempt a survey of the litera-
ture published during the last three years.   Thus, various individual
topics which have received attention during this period are now reviewed.

2. Ideal Fibre-reinforced Beams

An ideal fibre-reinforced material is first defined and then some recent theoretical results obtained when using this model are discussed.

A beam with straight fibres embedded in a matrix and aligned along the axis is considered transversely isotropic when the plane transverse to the fibres is a plane of isotropy. This beam is strongly aniso-tropic if Young's modulus associated with axial extension of the fibres is much larger than the values of Young's modulus in the transverse plane and the shear moduli of the matrix. In this circumstance the fibres may be idealised as inextensible with little sacrifice in accuracy. The composite is known as an ideal fibre-reinforced beam when, in addition, the material is assumed incompressible. This material idealisation is a continuum one, in which no distinction is made between the behavior of the fibres and the response of the matrix. The static elastic behavior of various ideal fibre-reinforced problems is discussed by Spencer (5) and simple theoretical solutions have been found in many cases.

Spencer (6) recently developed a theoretical procedure for studying the dynamic plastic structural behavior of ideal fibre-reinforced (strongly anisotropic) beams. Spencer employed the same general assump-tions which are customarily made to obtain the response of rigid-plastic beams made from an isotropic material (e.g. infinitesimal displacements, neglect of material elasticity and transverse wave propagation) and which under certain circumstances can lead to reasonable agreement with tests on experimental models (1). Spencer (6) examined the response of a beam of finite length initially traveling with a velocity V_0 which was

eventually brought to rest after suddenly striking a rigid stop. The solution of this particular problem can be transformed to give the behavior of a beam which is subjected, at the midpoint, to a constant velocity V_0, maintained for an infinite duration. This problem is in turn equivalent to the case of a beam of finite length which is struck by an infinite mass M traveling with a velocity V_0.

Theoretical solutions were developed in Reference (7) for the dynamic plastic structural response of various ideal fibre-reinforced (strongly anisotropic) beams with boundary conditions and external dynamic loadings which can be reproduced easily and reliably in a laboratory. The theoretical behavior of these beams was also compared to the corresponding dynamic response of beams which were made from a rigid perfectly plastic isotropic material. Generally speaking, it appears that the permanent transverse deflections and response durations of ideal fibre-reinforced beams loaded dynamically are less than the corresponding values for similar rigid perfectly plastic isotropic beams.

The theoretical predicions in References (6) and (7) were developed for a rigid linear strain-hardening ideal fibre-reinforced material. Spencer (8) re-examined the particular beam problem he studied in Reference (6) and presented a theoretical procedure which could be used for a wider class of strain-hardening materials. The theoretical predictions reported in Reference (9) for an ideal fibre-reinforced rigid plastic beam supported across a span of finite length and loaded impulsively, indicated that material strain hardening exercised an important influence on the magnitude of the permanent displacements and on the shape of the final deformed profile. Shaw and Spencer (10) have

examined the behavior of various beams struck by masses but simple theoretical results for some cases were only possible for a linear strain-hardening ideal fibre-reinforced material. No theoretical investigations appear to have been published on the dynamic plastic behavior of ideal fibre-reinforced plates and shells.

It is evident from the theoretical studies reported in References (7) and (9) that the response duration and permanent transverse displacements of ideal fibre-reinforced beams are significantly less than the corresponding quantities in "equivalent" rigid perfectly plastic isotropic beams. Thus, it appears that a potential for considerable weight savings may exist for energy absorbing systems made from materials characterised as ideal fibre-reinforced rigid-plastic. However, experimental investigations are required to establish if the ideal fibre-reinforced material model is valid for strongly anisotropic beams loaded dynamically. The particular cases examined in References (7) and (9) would offer attractive test arrangements for experimental work since similar rigid-plastic isotropic beams have been investigated by several authors.

3. Higher Modal Response of Beams

The previous interest in the modal response of plastic structures was associated with the development of approximate solution methods, and some recent work in this area is discussed in section 5 of this review. However, with the exception of a two-mass discrete model (11, 12), only primary or fundamental response modes have been examined. It is evident that an infinity of plastic modes is possible in a continuous structure, as in the elastic case. A knowledge of these mode shapes and associated accelerations can contribute to an increased understanding of the basic properties of rigid-plastic structures which deform in the plastic range as a result of dynamic loading. Moreover, the excitation of higher modal plastic deformations may find practical applications in the development of efficient energy absorbing devices.

Exact theoretical solutions for the first, second and third modal responses of fully clamped beams subjected to impulsive velocities having first, second or third modal shapes were presented in Reference (13). These theoretical predictions were compared to some permanently deformed profiles measured after a series of higher modal experimental tests on aluminum 6061 T6 511 beams, and it was concluded that geometry changes, or finite-deflections, exercised an important influence on the dynamic response, which agreed with previous studies on uniformly loaded beams (1). The simple theoretical procedure developed in Reference (14) was used to examine a first modal response of a beam, and confirmed that geometry changes were largely responsible for the discrepancy between the experimental results and the theoretical predictions developed using an infini-

tesimal theory.

The infinitesimal and finite-deflection analyses presented in Reference (13) were further generalised in Reference (15) to predict any symmetrical or antisymmetrical modal response of impulsively loaded, fully clamped, rigid perfectly plastic beams. The numerical elastic-plastic behavior of fully clamped beams subjected to impulsive "modal" velocity fields was also examined in Reference (15), using the spatial finite-element JET 3C computer program of Wu and Witmer (16, 17). It is evident, from Figures 4 to 6 in Reference (15), that the simple theoretical rigid-plastic procedure of Reference (14), which includes the influence of geometry changes, gives fairly good agreement with the numerical elastic-plastic finite-element results. This further confirms the accuracy and reliability of the generalised theoretical method, which was also compared to some experimental results on beams and rectangular plates in References (1) and (18) etc.

The magnitudes of the dimensionless transverse shear forces (Q/Q_0) (where $Q_0 = \sigma_0 H/\sqrt{3}$, σ_0 is uniaxial yield stress and H is beam thickness) were estimated from the bending moment distributions predicted by the JET 3C numerical elastic-plastic program, and are listed, for the first three modes, in Table 1 of Reference (15). It is evident from these numerical results that the ratio Q/Q_0 increases with increase in mode number, despite the fact that the dimensionless permanent transverse displacements are smaller for the higher modes. The largest numerical value of $Q/Q_0 = 0.35$ is associated with a third modal response. The excitation of higher modes in structures may generate even larger transverse shear forces. For example, integration of equation 8 in

Reference (15) and use of equations 9 and 17 gives
$Q(o) = \pm 3M_0 \left\{ 1 + (n - 1) \sqrt{2} \right\} / L$ for the value of the transverse shear
force at the supports of a beam undergoing a symmetrical modal response
with infinitesimal displacements, where $2n - 1$ is the mode number and
$M_0 = \sigma_0 H^2 / 4$. Thus, the transverse shear force can become large for
high modes, as expected, regardless of the value of the transverse dis-
placements. These observations would appear to justify further investi-
gations in order to establish the importance of transverse shear forces
on plastic yielding and to seek the influence of shear deformations on
the higher modal response of beams and other structures.

Although the excitation of pure modal responses is unlikely for
most practical problems, unless deliberately activated as in a specially
designed energy absorbing system, it is nevertheless apparent that the
behavior of complex structures loaded dynamically may involve complica-
ted deformation fields having some of the features of higher modal
responses.

4. Influence of Transverse Shear and Rotatory Inertia

Transverse shear forces can exercise a more important influence on the response of rigid-plastic beams loaded dynamically than in similar beams loaded statically (e.g. (19) and see section 5c of Reference (1)). Indeed, it has been demonstrated experimentally (20) and shown theoretically (18) that shear failures can develop at the supports of uniform isotropic beams loaded impulsively. The numerical results in Table 1 of Reference (15), which are discussed in section 3 of this review, also indicate that transverse shear forces may reach significant values in beams which undergo higher modal dynamic responses. Transverse shear forces also dominate the dynamic response of strongly anisotropic beams (6 - 10), which are discussed in section 2 here. The foregoing recent studies all involve beams loaded dynamically into the plastic range, but transverse shear effects would also play an important role in the dynamic plastic response of plates and shells.

It appears that there is considerable uncertainty in the literature on many aspects of the precise role of transverse shear forces, even for the yielding of rigid perfectly plastic beams loaded statically. Indeed it has been demonstrated that interaction curves relating bending moment (M) and transverse shear force (Q) are not proper yield curves and as further support to this view interaction curves for I-beams have been constructed which are not convex (21).

The role of transverse shear forces on the plastic yielding of beams was examined in a recent note (22), in which some justification was given for using convex yield curves for I-beams within the setting of engineering or classical beam theory. A suitable compromise from

an engineering viewpoint between the simple local (stress resultant)
and more rigorous non-local (plane stress, plane strain) theories may
be achieved for I-beams when using a local theory (e.g. Hodge (23)) with
a maximum transverse shear force based only on the web area.   It is
evident from Figure 5 in Reference (22) that Hodge's (23) revised theo-
retical results now provide an inscribing lower bound curve in the
$M/M_o - Q/Q_o$ plane which, because of its simplicity, might be acceptable
for many theoretical studies on beams.   Furthermore, the theoretical
predictions of Heyman and Dutton (21) and of Ranshi, Chitkara and
Johnson (24) and others are reasonably well approximated by a square
yield curve which has been used for solving various problems in dynamic
plasticity (e.g. 19).   In fact, Hodge's (23) revised results and a
square yield curve provide two simple methods for essentially bounding
the actual yield curve for an I-beam, as shown in Figure 5 of Reference
(22).

It was also pointed out in Reference (22) that Heyman's (21) objection
to the existence of convex yield curves on the grounds that $Q = dM/dx$ is
incorrect.

It is evident from Figure 6 in Reference (22) that a number of
local and non-local theories give similar curves in the $M/M_o - Q/Q_o$
plane for beams with rectangular cross-sections, so that one may select
whichever theory is the most convenient.

Symonds (19) has examined the influence of transverse shear forces
on the dynamic plastic response of an infinitely long beam struck by a
mass travelling with an initial velocity V_o.   Symonds simplified his
theoretical work with the aid of a square curve which relates the

values of bending moment (M) and transverse shear force (Q) required
for plastic yielding. More recently, Nonaka (25) used a similar theo-
retical procedure for a beam simply supported across a span of finite
length. The beam was subjected to a blast type loading (with a peak
value at t = O and decreasing monotonically with time t) distributed
uniformly across the entire span, and detailed theoretical results were
presented for the particular cases of impulsive loading, rectangular pulse
loading and exponentially decaying loading. Generally speaking, Nonaka's
observations lend further support to those of Symonds in that transverse
shear effects can be important for beams with non-compact cross-sections
regardless of the type of dynamic loading, while they are important for
compact beams when subjected to dynamic pressures which are much larger
than the corresponding static plastic collapse pressure (e.g. impulsive
loading).

No restrictions were placed on the amount of shear sliding at the
stationary plastic "hinges" which developed in the theoretical analyses
presented in References (19) and (25). However, it is clear that com-
plete severance has occurred when the amount of shear sliding equals the
beam thickness, as remarked in Reference (18). Thus, it is necessary to
ensure that this mode of failure does not intervene and control the res-
ponse of a particular beam rather than the theoretical results presented
in References (19) and (25).

It appears that the influence of rotatory inertia has been neglected
in all analytical investigations on the dynamic plastic response of struc-
tures. This situation prevailed despite the many studies which have explored
the role of rotatory inertia in various dynamic elastic problems. Thus,
an exact theoretical procedure, which retained the influence of rotatory

inertia and transverse shear forces on the dynamic plastic behavior of
beams, was developed in Reference (26).   The behavior of a long beam
impacted by a mass and the impulsively loaded simply supported beam prob-
lem of Nonaka (25), were examined using a square yield criterion.

The theoretical predictions for the particular parameters considered
in Reference (26) indicated that rotatory inertia barely influenced the
dynamic plastic response of a long wide-flanged I-beam impacted by a mass,
while a more noticeable effect was observed for a simply supported wide-
flanged I-beam loaded impulsively.   As expected, rotatory inertia exer-
cised the greatest influence on beams with rectangular cross-sections.
The largest reduction observed in the maximum dimensionless transverse
displacements was approximately 11% for the particular calculations repor-
ted in Reference (26).   It appeared from this study that the effect of
rotatory inertia on the dynamic plastic response of beams is sensitive
to the kind of boundary conditions and type of loading.

The influence of transverse shear forces and rotatory inertia on the
dynamic plastic response of beams with non-linear yield curves is examined
with the aid of a numerical procedure in Reference (27).

5. Approximate Methods of Analysis

The last* review on the dynamic plastic response of structures in the Shock and Vibration Digest (1) focussed largely on the influence of finite deflections, or geometry changes, and material strain rate sensitivity. More recently, Symonds and Chon have examined these non-linear topics from a different viewpoint in References (28) to (32).

In Reference (28), Symonds observed that the inhomogeneous nature of the strain rate sensitive constitutive equations complicates the theoretical mode type solutions and bounding methods. Thus, Symonds explored the possibility of replacing the inhomogeneous relations by simpler homogeneous viscous expressions. It turned out that replacing a rigid viscoplastic constitutive equation with a homogeneous viscous representation did simplify the theoretical analyses, and the best agreement was found when matching the initial stresses and initial slopes of the dimensionless stress-strain rate inhomogeneous exact and homogeneous viscous curves. The homogeneous matched viscous constitutive relations constructed in this way were used in the theoretical work presented in References (29) to (32).

Symonds and Chon (29, 30) developed an upper bound theorem to estimate the permanent displacements of strain rate sensitive structures loaded impulsively. They used the theorem of minimum potential energy as a starting point and utilised the extremal path concepts introduced by Ponter (33) in order to obtain well defined functions of specific strain energy and specific complementary strain energy, when the influence of

*The last review at the time of writing this article. Reference (128) has since been published.

finite-deflections was retained in the basic equations.   It turned out
that the extremal paths for homogeneous viscous relations between stress
and strain rate were extremely simple (see equation 6 of Reference (29)).
The theorem was developed by comparing the response of the dynamic
problem with the behavior of the same structure subjected to a static
loading which produces stresses and strains following an extremal path.
If the static loading is taken as a concentrated force, then an upper
bound on the displacement at the same location for the dynamic problem
can be found (equation 15b of (29)), when the total strain energy of
the static problem equals the initial kinetic energy for the dynamic prob-
lem (equation 15a of (29)).   Unfortunately, this theorem requires
knowledge of the response duration, for which no rigorous bounds exist,
when finite-deflections and material strain rate effects are retained in
the basic equations.   However, the authors observed that the theoretical
predictions were not very sensitive to the actual value of the response
duration, so that either Martin's time bound (34) or the approximation
developed in section 5 of Reference (29) could be used.

Symonds and Chon (29, 30) used their theorem to examine simple, one
degree of freedom and two degrees of freedom structural models and
obtained upper displacement bounds which, in the case of the one degree of
freedom model, were quite close to the exact maximum displacements.   The
upper bound theorem was also used to study an impulsively loaded fully
constrained beam with a sandwich cross-section, and the theoretical pre-
dictions for the maximum permanent transverse displacements were com-
pared to the corresponding experimental results (35) in Figure 8 of
Reference (29).   The upper bound predictions are quite accurate at the

lower impulse levels but lie significantly above the experimental results for large impulses, which are responsible for maximum permanent trans- verse displacements greater than about five beam thicknesses. Unfortunately, the predictions of the theorem were not presented for a rigid perfectly plastic material, so that it is not known whether approximations associated with geometry changes or the strain rate sensi- tive relations, or both, require refinement to improve the estimates at the larger impulse levels.

Symonds and Chon (31, 32) have also examined mode approximation techniques for strain rate sensitive structures, when the influence of finite-deflections, or geometry changes, were retained in the basic equa- tions. The modal responses considered in section 3 of this article were associated with structural problems, in which the external loading was responsible for an exact structural response with a velocity profile having a time-independent shape (i.e. modal response). However, the mode approximations in References (31) and (32) follow the spirit of Reference (36) in that the behavior of any dynamic problem may be approxi- mated with a modal response. If the same modal form remains valid throughout the entire structural response, then it is called a "permanent"* mode form solution. On the other hand, a sequence of mode form solu- tions may be required in certain classes of problems (e.g. when geometry changes are retained in the basic equations). It is clear that the initial velocity field in a theoretical solution using a mode approxima-

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*In some earlier work these were called stationary mode forms.

tion is not the same as the initial velocity distribution of the corresponding impulsive loading problem, except in exceptional circumstances (as in section 3 here). However, Martin and Symonds (36) developed a criterion for judiciously selecting the magnitude of the modal velocity field in order to minimise the error due to the different initial conditions. More recently, Chon and Martin (11) have examined the mode approximation method for a simple two degrees of freedom viscoplastic beam. This theoretical work was developed for structures which undergo infinitesimal displacements and no extension has yet been made to incorporate the influence of finite-displacements.

Symonds and Chon (30) studied a two degrees of freedom beam problem, which consisted of two masses connected by massless links, and obtained a stationary mode solution when the end supports were free to move inwards. However, if the supports were restrained axially, then it became necessary to use an iterative method to obtain a sequence of mode solutions.

The mode approximation procedure is used in Reference (31) to examine a strain rate sensitive fully clamped circular plate which undergoes large transverse deflections. The plate was assumed to have a sandwich cross-section which was made from a homogeneous viscous material. An iterative solution procedure was developed which required about 5 to 10 cycles in order to achieve no more than a 5 per cent difference between two successive iterations of the mid-point velocity at a given time. This numerical procedure was repeated at about 13 different time steps to generate the theoretical estimates for the final permanent transverse displacements which are presented in Figures 3 and 4 of Reference (31).

The various approximations in the theoretical procedure are all con-
servative in the sense that they should lead to overestimates of the
transverse displacements. However, the theoretical predictions lie
below the corresponding experimental results of References (37) to (39),
although they do provide reasonable estimates. It is not clear why this
situation prevails, but some suggestions are given in Reference (31).

The mode approximation technique was also employed in Reference (32)
to study the dynamic behavior of circular plates subjected to axisymmetric
pressure pulses when geometry changes were disregarded (i.e. infinitesi-
mal displacements). It was found that stationary mode solutions only
existed for impulsive loadings, or when the shape of the external pres-
sure distribution was the same as the modal shape of the transverse dis-
placement field.

Bodner and Symonds (40, 41) have recently presented some experi-
mental results which were obtained from a test program conducted on
strain rate sensitive frames and fully clamped circular plates subjec-
ted to impulsive velocities which produce large permanent displacements.
These experimental results are compared in References (42) and (43) with
the theoretical predictions of the displacement bound and mode approxi-
mation methods which are discussed in Reference (30).

As expected, the upper bound theorem in References (29) and (30) and
the mode approximation technique in References (31) and (32) are con-
siderably more difficult to use than simple rigid perfectly plastic
methods and even more difficult than the comparatively simple procedure
to cater for the influence of finite-displacements which was developed in
Reference (14) and discussed in Reference (1). However, the methods in
References (29) to (32) do provide some information about certain features

of the structural response which would be much more expensive to obtain when using a wholly numerical procedure. Perhaps, more importantly, they do provide insight into some characteristics of plastic structural dynamics.

It should be remarked that Ploch and Wierzbicki (44) have constructed an alternative theorem for obtaining upper bounds on the permanent displacements of impulsively loaded rigid perfectly plastic structures with large displacements. Ploch and Wierzbicki used this theorem to predict reasonably good values for the maximum permanent transverse displacements of a fully clamped beam subjected to a uniformly distributed impulsive velocity field.

The fundamental characteristics of dynamic plastic mode solutions for structures which undergo infinitesimal displacements have been examined from a variational viewpoint in References (45) to (47). Erkhov (45) studied the dynamic plastic behavior of a simply supported rigid perfectly plastic shallow cap subjected to a uniformly distributed pressure with a rectangular pressure-time history. It turns out that Erkhov's theoretical results for the permanent displacement profile (equation 3.9) for intermediate pressures is identical to the theoretical solution in Reference (48) (equation 18), except for a difference in the static collapse pressures which is related to the use of different yield criteria. The authors of References (46) and (47) respectively used a two degrees of freedom system and a simply supported circular plate to illustrate their theoretical work.

Wierzbicki (49) developed a simplified strain rate sensitive constitutive relation and used it to demonstrate that an eigenfunction

expansion method could then be employed to examine the dynamic behavior

of plastic continua when the displacements remained infinitesimal.

Maier and Corradi (50) have derived an upper bound theorem for the

dynamic infinitesimal displacements of elastic-plastic continua using

the principle of virtual work and Drucker's stability postulate.   The

final form of this theorem contains only quantities which are either

known when motion commences or are related to a solution of the same dyna-

mic problem when considered wholly elastic.

6. Rapidly Heated Structures

It is evident that large neutron pulses may cause a temperature gradient to develop through the thickness of a structure made from a fissile material. This temperature gradient may give rise to curvature changes of beam, plate and shell structural members in addition to various other effects. The structural response is governed by the equations of dynamics when these curvature changes develop in a sufficiently short time. Thus, the dynamic structural response may be estimated with the aid of rigid-plastic methods of analysis when the temperature accelerations are sufficiently large.

Parkes (51) examined the dynamic infinitesimal displacement response of a free rigid-plastic beam subjected to a time-dependent and spatially-independent thermal curvature (χ_T). Parkes observed that various types of behavior were obtained which depended on the rapidity of heating (or $\ddot{\chi}_T$). It was found that a free beam remained rigid* for sufficiently small values of $\ddot{\chi}_T$, while for larger values a discrete plastic hinge developed and became an expanding and contracting plastic hinge (zone) at still larger values of $\ddot{\chi}_T$.

It turns out that the behavior of a free beam with a temperature-independent plastic moment and subjected to $\ddot{\chi}_T$ with a sinusoidal** temporal form (equation 22 of Reference (51)) is quite complicated, as

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*The beam deforms due to the thermal curvature χ_T but the associated maximum bending moment is less than the fully plastic bending moment.

**This particular form is characteristic of neutron heating in a pulsed reactor.

indicated in Figure 3 of Reference (51). For example, the dynamic response associated with a large value of $\dot{X}_T$ commences with a rigid phase which is followed by another phase with a positive central plastic hinge. An expanding positive plastic hinge (zone) then develops and is followed by three further phases, which are characterised by a positive central plastic hinge, a contracting plastic hinge, and a negative central plastic hinge before returning to a rigid phase. The sequence of events depends on the magnitude of $\ddot{X}_T$ as indicated in Figure 3 of Reference (51) and is undoubtedly also a function of the form of $\ddot{X}_T$ which has not yet been explored.

In a more recent publication, Parkes (52) used the general theoretical work in Reference (51) to examine the dynamic rigid-plastic behavior of a rapidly heated cantilever beam subjected to the same form of $\ddot{X}_T$ considered in Reference (51). Again the exact theoretical solution is complicated, even though the displacements remain infinitesimal and the plastic moment is temperature-independent. However, Parkes (52) observed that the approximate theoretical analysis of a so-called strong beam, with all the deformation restricted in a hinge at the root of a cantilever, was much simpler and predicted surprisingly accurate values for the final displacements after the relief of the thermal curvature.

Wisniewski (53) examined the dynamic structural behavior of an aluminum 6061 T6 rectangular plate subjected to an X-ray deposition. The surface of the plate exposed to the X-ray deposition is heated almost simultaneously to high temperatures, which causes some of the material to melt and blow-off. This blow-off causes an impulsive load, which creates a stress wave that propagates through the plate thickness towards

the rear surface.   Simultaneously, the material below the exposed sur-
face is heated and a compressive stress wave is induced which is reflec-
ted as a rarefaction tensile wave from the rear surface.   It is evident
that the possibility of spalling for sufficiently large stresses must
therefore be considered in a study.

Wisniewski (53) simplified the posed problem and then used an
available computer program to predict the material behavior during the
first few microseconds of the response.   The numerical predictions were
then used to generate the input for an elastic-plastic finite-difference
scheme (54) to predict the structural response, which involves response
durations of the order of milliseconds, as discussed in Reference (1).

7. Fluid-structure Interaction

In a recent review published in the Shock and Vibration Digest (55), Krajcinovic distinguished between the steady-state and transient behavior of structures interacting with a fluid, and then proceeded to discuss the general features of transient interaction problems with constant and variable wetted surfaces. The theoretical behavior of a transient interaction problem is simplified considerably when the structural material is idealised as rigid-plastic. The theoretical predictions of such analyses are particularly useful when the ultimate performance of a structure is of primary concern, provided the usual restrictions associated with this type of an approximation are satisfied (1 - 4, 55).

Krajcinovic (56) examined the dynamic response of a simply supported rigid-plastic beam, which rested on a semi-infinite pool of incompressible, irrotational and inviscid fluid. The governing equations were formulated for a beam subjected to a time-dependent external pressure which caused infinitesimal transverse displacements. The greater part of Reference (56) focussed on the difficult task of evaluating the fluid back pressure and the virtual mass associated with a simple triangular deformation mode for the beam without any travelling hinges. Krajcinovic then examined a simply supported beam subjected to a uniformly distributed pressure and found that a single mode transverse deformation profile remained valid provided the magnitude of the external dynamic pressure was less than about three times the corresponding static collapse pressure (see equation 47 of Reference (56)). As expected, the perma-

nent transverse displacements are less than those which would be obtained in vacuo.

Krajcinovic (57) also examined the dynamic behavior of a simply supported circular rigid perfectly plastic plate resting on a potential fluid. The response due to a uniformly distributed external pressure with a rectangular shaped pressure time history was obtained for a conical transverse displacement profile. It turned out that a conical displacement profile could be used for external pressure pulses with magnitudes up to somewhat larger than twice* the corresponding static collapse pressure. The theoretical predictions for the impulsive velocity loading case were also presented in Reference (57). However, these results must remain in doubt until it is established that a statically admissible generalised stress field can be associated with a conical displacement profile. This is likely to be a fruitless enterprise because it is known that travelling plastic hinges develop during the response of the same problem in vacuo.

It is evident from the foregoing comments that equations 47 and 60 in References (56) and (57), respectively, restrict the external dynamic pressures to low magnitudes which are only a few times larger than the corresponding static collapse pressures. Larger external pressures would probably give rise to travelling plastic hinges, which would complicate the calculation of the fluid back pressure and virtual mass.

The non-linear influence of finite transverse displacements, or geometry changes, would exercise an important effect on the response when

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*Equation 60 gives a ratio of 2 when in vacuo and 2.346 for a steel plate with a radius to thickness ratio of 25 resting on water.

the maximum permanent transverse displacements exceeded the correspond-
ing structural thickness, as discussed in References (1) and (14) for
beams and plates in vacuo and exposed to dynamic loads on one surface.
The theoretical results in Figure 4 of Reference (58) illustrate the
important strengthening influence of membrane forces which were developed
in a simply supported circular plate subjected to a static pressure dis-
tributed over the entire surface on one side of a plate, and another
static pressure distributed within a circular region on the opposite sur-
face.

The theoretical solutions in References (56) and (57) were simpli-
fied by neglecting the generation of waves on the surface of the fluid.
However, in a practical beam or plate problem with a single mode trans-
verse deformation profile, it is inevitable that waves would be genera-
ted on the fluid surface outside the supports when the fluid is assumed
incompressible.   Nevertheless, it is possible to retain the incompressi-
bility assumption without the generation of surface waves by using more
complex transverse displacement fields (e.g. a modification of the third
modal velocity fields examined in References (13) and (15) and discussed
in section 3 here).

Duffey (59) examined the transient response of viscoplastic spheri-
cal shells submerged in a fluid.   The inner surface of a shell was sub-
jected to a spherically symmetric impulsive velocity which produced a
spherically symmetric structural response.   Duffey considered the inviscid
fluid to be compressible and used the classical wave equation to evaluate
the fluid pressure.   Furthermore, the wall thickness of the shell was
assumed to be sufficiently thin so that the material throughout the entire

shell could pass simultaneously from an elastic state to a plastic state. With these simplifications, Duffey was able to examine the influence of material elasticity (including unloading), material strain hardening, and material strain rate sensitivity.   One interesting feature of the results presented in Figure 5 of Reference (59) is the significant error associated with the linearisation of the strain rate sensitive relations.

The theoretical solutions in References (56), (57) and (59) were developed for structures with one side loaded by a dynamic load, while the other side was in contact with a fluid (e.g. an internal explosion in the hull of a ship).   Sometimes, structures are subjected to a disturbance which travels through a fluid from a distant source (e.g. an external explosion acting on the hull of a ship (60)).   Other types of practical problems may involve the impact of a structure on a fluid (e.g. slamming of ships and marine vehicles), or the impact of a fluid on a structure (e.g. water wave impact on a barrier or offshore platform). Simple rigid-plastic methods were developed in References (61) to (63) to estimate the damage sustained by ships and marine vehicles from severe slamming and bow impacts.   These theoretical methods gave surprisingly good agreement with the corresponding experimental results and could be further developed to examine various other problems.

8. Dynamic Plastic Buckling

The work described in the previous sections focuses on the inelastic behavior of structures which have a "stable" response when subjected to dynamic loads. However, dynamic plastic buckling or unstable behavior, which is characterised by wrinkling as in static buckling, can occur when certain structures are acted on by large external loads.

A brief literature review on the dynamic plastic buckling of rods, flat plates, cylindrical shells and spherical shells was presented in section 5e of Reference (1). It appears that the dynamic plastic instability of all the structural problems so far investigated stems from the growth of small imperfections in the otherwise uniform initial displacement and velocity fields. Unlike classical static buckling analyses, a distinct value of the dynamic load which causes structural instability is not predicted by these theoretical analyses. Rather, an expression is obtained which indicates how the displacement profile of a structure grows with time for different levels of dynamic load. Buckling is said to occur when the dynamic load reaches a threshold or critical value which is associated with the minimum unacceptable or maximum acceptable deformation, the magnitude of which is defined arbitrarily.

The dynamic plastic buckling of a cylindrical shell made from a rigid linear strain-hardening material and subjected to a uniformly distributed, almost axisymmetric external impulsive velocity field, was examined in Reference (64). A particularly simple solution was found for an infinitely long cylindrical shell which offered the advantage that various characteristics of the response could be examined analytically.

For example, the dynamic plastic buckling of a long cylindrical shell turns out to be more sensitive to initial imperfections in the profile than to imperfections in the initial velocity field. This is fortuitous because it is usually easier to control imperfections in the initial shape than any imperfections in the initial velocity field. Moreover, equations 20 and 21 in Reference (64) indicate that the greatest amount of scatter might be expected to occur in the experimental critical mode numbers of long cylindrical shells with large radius to thickness ratios and/or small values of the material parameter β_1 (ratio of tangent modulus to average flow stress). Furthermore, equation 27 in Reference (64) indicates that local elastic unloading is more likely to occur for shells with the larger radius to thickness ratios and/or smaller values of the material parameter β_1.

The theoretical predictions for the dominant behavior, critical mode numbers, and threshold impulses from all known previous studies on the dynamic plastic buckling of cylindrical shells and rings subjected to external impulsive velocities are summarised in section 3 of Reference (64). In addition, experimental results are also presented from a test program on hot rolled mild steel and aluminum 6061 T6 rings, which were subjected to axisymmetric external impulsive velocity fields. These experimental values are compared, in Figures 5 to 14, in Reference (64), with all known experimental results and theoretical predictions for the dynamic plastic buckling of rings and cylindrical shells. It is evident from these Figures that the various theoretical predictions are widely divergent, some giving good agreement with the corresponding experimental values, while others are unsuitable.

Generally speaking, the simple theoretical predictions for the permanent radial displacements of rigid perfectly plastic rings subjected to an axisymmetric velocity field (equations 28 and 33 of Reference (64)) agree reasonably well with the permanent average radial displacements recorded in the experimental tests, provided any material strain rate sensitivity is catered for, as suggested by Perrone (65) and demonstrated for shells in Reference (66).

The critical mode numbers observed during the current tests are compared in Figures 9 and 10 of Reference (64) with the results of all previous relevant experimental investigations known.   The results are reasonably consistent, notwithstanding the differences in yield stresses of the materials, experimental techniques, and despite the fact that the buckled profiles of cylindrical shells and rings are irregular.   Generally speaking, the experimental critical mode numbers increase with increase in the length to radius ratio and with increase in the radius to thickness ratio.

The experimental results in Figure 11 of Reference (64) indicate that respect of the threshold impulses estimated by Florence and Vaughan (67) (equation 49 of Reference (64)) ensures that the permanent wrinkles in the deformed profiles of the rings remain small.   However, the experimental results in Figure 12 (64) demonstrate that the ratio of wrinkle (buckle) amplitude to average permanent radial displacement decreases with increase in the impulse magnitude.

The manner in which initial geometric imperfections of the rings influences the wrinkle amplitude is explored in Figure 14 of Reference (64).

The general features and characteristics of iso-damage curves in a dimensionless peak load-impulse space were discussed by Abrahamson and Lindberg (68), who found them a convenient representation for theoretical predictions and experimental results on the dynamic response of various structures subjected to pulse loads as distinct from oscillatory loads. Moreover, Abrahamson and Lindberg observed that an iso-damage curve for the dynamic buckling of a simply supported cylindrical shell under uniform lateral pulse loads is not very sensitive to the amplification factor (factor by which the initial imperfections grow during the response) or the pulse shape. Iso-damage curves are also plotted in Reference (68) for the dynamic plastic response of beams and circular plates subjected to transverse dynamic loads which are responsible for stable behavior and infinitesimal displacements. It might also be remarked that iso-damage curves have been presented in Figure 5 of Reference (62) for rectangular plates which retain the important influence of finite transverse displacements, or geometry changes.

Florence and Abrahmson (69) observed that the stability of cylindrical shells and rings subjected to large external impulsive velocities improved during deformation when the increase in wall thickness was taken into account. This led Florence and Abrahamson to define a critical impulsive velocity which is associated with a specified acceptable amplification factor. Thus, impulsive velocities larger than the critical value produce acceptable departures from circularity of a cylindrical shell, while impulsive velocities smaller than the critical value are responsible for unacceptable damage. It is interesting to observe, from the general trend of the experimental results reported in

Figure 12 of Reference (64), that the ratio of the wrinkle (buckle)
amplitude to the average permanent radial displacement of a ring decreases
with increase in impulse.

Florence and Abrahamson (69) formulated the governing equations for
a ring which was made from a rigid linear viscoplastic material, and
studied the particular case when the impulsive velocity remained con-
stant during the collapse of a cylindrical shell onto its longitudinal
axis.   The governing equations were solved numerically and various
features of the response are presented in Figures 4 to 11 of Reference
(69).   These results indicate that the presence of linear viscoplasticity
drastically reduces the amplification factors and the preferred mode
numbers, and thereby contributes to the enhanced stability of a cylindri-
cal shell.

Lee explored the bifurcation and uniqueness of elastic-plastic
continua loaded dynamically from a general viewpoint in Reference (70),
because random initial imperfections, or perturbed motion, may be
insufficient to describe the dynamic plastic buckling of some structures.
Lee further examined this subject in Reference (71) and developed a quasi-
bifurcation criterion for the stability of elastic-plastic continua loaded
dynamically.   A quasi-bifurcation of motion develops at a certain time
t_{cr} when a nontrivial perturbed motion exists, which makes the functional
defined by equation 51 in Reference (71) an extremum.   No applications
of this theorem have yet been published, but Lee claims to have used it
with success to describe the dynamic plastic buckling of rods subjected
to axial loads.   Lee showed that his dynamic quasi-bifurcation criterion
reduces to Hill's bifurcation theory for quasi-static loads.

9. Numerical Studies

A number of computer programs are available for studying the dynamic plastic behavior of various structures. A brief review of some of these finite-difference and more recent finite-element numerical schemes is given in section 5f of Reference (1).

The Aeroelastic and Structures Research Laboratory at the Massachusetts Institute of Technology has continued its active development of both finite-difference and finite-element numerical procedures for the dynamic behavior of a broad class of structural problems. In Reference (72), Leech, Witmer and Morino used the finite-difference computer code PETROS 3 to study the dynamic response of a variable-thickness, double-layer, clamped-ended, elastic-plastic conical shell subjected to a frontal cosine external pressure pulse. This computer program has now been expanded in References (73) and (74) to include various new features which are largely related to thicker shells, thin non-Kirchhoff soft-bonded, and/or honeycomb shells. The impressive range of capabilities of the PETROS 4 finite-difference computer program is listed in Table 1 of Reference (74).

Wu and Witmer have continued to develop their finite-element numerical scheme for the dynamic response of various structures. Wu (75) demonstrated that his numerical predictions for the deformed profile of a fully clamped impulsively loaded rectangular plate agreed quite well with the corresponding experimental results reported in Reference (76). Wu and Witmer (77) further developed the spatial finite-element and temporal finite-difference scheme in order to study a broader class of structural problems. They found somewhat better agreement with experimental results recorded during the dynamic elastic-plastic response of a cylindrical

shell panel than had been reported previously for a computer code using a finite-difference procedure. The numerical predictions for the non-linear transient responses of geometrically stiffened cylindrical panels and rings also compare favourably with the corresponding experimental results reported in References (78) and (79), respectively. This work demonstrates that a finite-element scheme provides efficient and accurate predictions for the transient behavior of structural problems which involve large deflections and elastic-plastic material behavior.

The numerical schemes of Wu and Witmer were used in Reference (15) to examine the dynamic response of strain rate insensitive elastic perfectly plastic fully clamped beams subjected to impulsive velocity fields distributed with first, second and third modal forms. The maximum permanent transverse displacements for the three modal forms were similar to the corresponding theoretical predictions of a simple rigid perfectly plastic theory which included the influence of finite transverse displacements (14). A number of assumptions which are customarily made in theoretical studies in this general area were explored in Reference (15) with the aid of the computer program. Generally speaking, the in-plane displacements turn out to be one order of magnitude smaller than the associated lateral or transverse displacements, while it was shown that the usual procedure for estimating energy ratios (80) is conservative, at least for the problems considered.

In order to circumvent the considerable expense usually associated with numerical finite-element studies, especially for dynamic non-linear problems, Kawai has developed an alternative numerical scheme, in which a structure is replaced by an "equivalent" system of small rigid bodies

connected to "springs" distributed over the contact areas between neighbour-
ing bodies.   This numerical scheme has been used to predict the dynamic
elastic-plastic response of a beam and a square plate in References (81)
and (82), respectively.   However, the important influences of finite
transverse displacements, or geometry changes, and material strain rate
sensitivity (1) were not retained in these calculations, so that further
work is required in order to demonstrate the accuracy and appraise the
cost of the proposed method for more complex problems.

Many other computer programs have been written in order to examine
the dynamic plastic response of either particular structural components
or more general structural geometries.   A few of these papers which
have reached the writer's attention over the last few years will now be
briefly mentioned.

The authors of References (50) and (83) to (85) have developed some
quite general theoretical principles and formulated various numerical
procedures for predicting the dynamic plastic behavior of structures.
Viscoplastic and large displacement effects are considered in Reference
(83), while material elasticity, material work hardening, and geometry
changes are retained in the basic equations used in Reference (84).   The
studies in References (50) and (85) are restricted to infinitesimal dis-
placements, but Reference (50) retains elastic effects, while the influence
of material strain hardening and strain rate effects may be taken into
account in Reference (85).   Some beam and frame impact problems are
examined in Reference (85).

Erkhov (45) has formulated the dynamic infinitesimal response of
rigid perfectly plastic structures as a linear programming problem which

may then be solved using the simplex method.   Erkhov used his procedure
to investigate the dynamic plastic behavior of circular and square
simply supported plates subjected to uniformly distributed pressure pulses
with a rectangular pressure-time history.

Ni and Lee (86) used a numerical scheme based on their minimum
principle for dynamically loaded elastic-plastic continua with finite-
deformations to predict excellent agreement with the numerical and experi-
mental results of Leech et al. (87) on cylindrical shell panels, the
experimental results in Reference (76) for rectangular plates, and the
theoretical predictions in Reference (88) for cylindrical shells.

Bieniek, Fumaro and Baron (89) have sought ways in which to simplify
numerical studies on the dynamic large displacement response of elastic-
plastic stiffened shells with arbitrary geometry.   In Reference (90),
Bieniek also discussed various numerical difficulties associated with
investigations into the dynamic buckling behavior of elastic-plastic struc-
tures.

One of the difficulties associated with all theoretical and numerical
studies on the dynamic inelastic behavior of structures is the paucity
of information on the constitutive equations for materials, especially in
the dynamic regime.   The influence of material elasticity, material
strain hardening and strain rate sensitivity were discussed briefly in
sections 5a, 5d and 5f of Reference (1), respectively.   It was remarked
in References (1), (62), (66) and (80) that the multi-dimensional con-
stitutive equations are invariably constructed using the properties
observed during uniaxial tests.   Moreover, the form of the multi-
dimensional constitutive equations for elastic-plastic materials is still

not clear, even for static problems (91).   This shortcoming is compounded
for dynamic problems because scarcely any experimental information exists
for strain rate history effects, or combined loadings (92).

Bodner and Partom (93 - 95) have developed a constitutive equation
which is not based on the usual concept of a yield surface.   The incre-
mental constitutive equations are functions of state variables and the
current geometry which can cater for all history and memory effects.
Bodner and Partom used the concepts of dislocation dynamics to provide a
physical basis for their constitutive equations which they claim are
ideally suited for numerical schemes because no special conditions are
required to distinguish between loading and unloading paths.   Recently,
Sperling and Partom (96) have developed a numerical finite-difference
procedure based on the Bodner-Partom constitutive equations and examined
the dynamic elastic-viscoplastic large deflection behavior of a beam.

10. Miscellaneous Comments

10(a) Introduction

An attempt was made in the previous sections of this review to survey the literature published on individual topics during the last three years. It was not possible to do this completely satisfactorily in section 9 because numerical methods really require a separate article to give adequate justice to the large amount of activity in this particular area. There are many other theoretical studies and practical dynamic problems which involve material inelasticity. Some of this work, which crossed the writer's desk during the last three years, is now mentioned briefly. This section, then, is to be regarded as a sampling of the many current activities on the dynamic plastic response of structures and is not intended to be comprehensive.

10(b) Experimental Studies

Forrestal and Wesenberg (97, 98) performed some dynamic experimental tests on simply supported beams which were made either from aluminum 6061 T6 or from mild steel. The beams were subjected to a short duration magnetic pressure pulse with a half-sine wave shape, using an experimental arrangement similar to that used previously for rings (99). Forrestal and Wesenberg (97, 98) also developed simple approximate elastic-perfectly plastic and elastic-viscoplastic theoretical procedures for predicting peak transverse displacements which are in excellent agreement with the corresponding experimental values.

Experimental studies on the dynamic plastic behavior of various other beams have been reported in References (13, 96) and (100).

Bodner and Symonds (40) conducted an experimental investigation on simple plane metal frames loaded dynamically. Experiments on the dynamic plastic behavior of fully clamped circular plates were reported in References (38) and (41). Witmer et al. also examined the dynamic plastic response of a freely suspended cylindrical shell in Reference (38). An experimental investigation into the dynamic plastic buckling of circular rings was reported in Reference (64). This article also gives adequate justice to the large amount of activity in this particular reviews the literature containing experimental work on the dynamic plastic buckling of rings and cylindrical shells.

An experimental investigation into the structural characteristics of high explosive containment in cylindrical vessels was conducted in References (101) and (102). Simple rigid-plastic theoretical procedures were also developed by these authors, who found that they provided reasonable estimates for the permanent radial deformations of the walls of cylindrical vessels, either with or without end caps.

10(c) Theoretical and Numerical Studies

Many theoretical and numerical studies were discussed in the previous sections. A few additional articles are now briefly mentioned.

Krajcinovic (103) derived a theoretical rigid perfectly plastic solution for the dynamic infinitesimal displacement response of a simply supported beam subjected to a uniformly distributed dynamic load with an arbitrary pressure-time history.

Youngdahl (104) examined the dynamic plastic behavior of a rigid perfectly plastic hexagonal frame subjected to an internal pressure pulse with an arbitrary shape. The important influence of finite dis-

placements, or geometry changes, was retained in the governing equations.

The behavior of a multi-layered spherical vessel subjected to a spherically symmetric intermittent internal pressure pulse was examined in Reference (105). The vessel consisted of N concentric unsupported spherical shells made from an elastic-linear work hardening material and separated by evacuated gaps. The pressure pulse caused the inner layer to move outwards and strike the second layer. These two layers moved outwards together until they separated due to wave interactions. The second layer then struck the third layer and the process was repeated.

Lepik and Mroz (106) used a mode approximation procedure to obtain the optimal design of rigid-plastic structures subjected to dynamic loads. The objective was to seek the design which gave the minimum permanent displacements of a structure with a given constant volume of material. The particular case of a stepwise constant thickness beam subjected to a uniformly distributed pressure-pulse with a rectangular pressure-time history was examined in some detail. It was found that the maximum permanent deflection of an optimal two-step (per half span) beam was one-half the corresponding permanent deflection of a uniform beam having the same volume. The impulsive loading of beams and circular plates was also examined in Reference (106).

Menkes and Opat have continued their work, discussed in section 1 of Reference (1) and in Reference (18), on the dynamic fracture of structures subjected to very high pressures for very short times. Menkes and Opat presented in Reference (107) the theoretical foundations of a finite-element numerical procedure for simple structures which undergo large deformations and localised ruptures.

10(d) Collision Protection of Vehicles

Many articles have been written on the collision protection of various air, land and water vehicles, and some of this literature is reviewed in References (4), (63), (80), (108) and (109). A few of the recent theoretical and experimental studies on the dynamic inelastic response of various components of interest for vehicle crashworthiness are discussed briefly in this section.

The dynamic axial collapse behavior of thin-walled steel box sections was examined in References (110) and (111), while the response of circular tubes, corrugated tubular sections and beam-columns subjected to dynamic axial loads was investigated in References (112) to (114), respectively. The review article by Thornton and Dharan (115) also contains some comments on the dynamic axial buckling of structures.

Shieh (116) developed a general purpose computer program for the large displacement dynamic response of elastic-viscoplastic plane frames. The numerical predictions of this computer program agreed reasonably well with the experimental behavior of a steel plane frame dropped onto a narrow rigid pole obstacle. McIvor and Anderson (117) have also reported a numerical and experimental investigation into the inelastic response of a frame striking a pole. McIvor et al. (118, 119) studied the large deformations of frames loaded statically to provide insight into the behavior of structural members in vehicles.

Garnet and Armen (120) used a finite-element procedure to examine the mechanics of impact and rebound of an elastic-linear work hardening rod subjected to axial forces, which are applied and removed periodically, and which hits a rigid wall at right angles. Lush and Witmer (121)

conducted some experimental tests to study the impact characteristics
of rodlike missiles striking either a flat rigid barrier or the mid-
span of a fully clamped beam. The missiles had a crushable forebody
made from a semi-rigid polyurethane foam and a comparatively rigid alumi-
num afterbody.

The theoretical and experimental behavior of inversion tubes and
rolling torus load limiters under static and dynamic axial loads was
examined in References (122) to (124).

Tong and Rossettos (125) used a modular concept and developed a
numerical procedure to estimate the structural deformations sustained by
vehicles during collisions.

11. Concluding Remarks

It was remarked in sections 1 and 10(a) that material inelasticity
plays an important role in many dynamic structural problems which arise
in a number of diverse areas. The studies reported in References (104)
and (126) were motivated by the nuclear engineering industry, References
(61 - 63, 109 and 127) are related to ship and marine vehicle design,
and the References quoted in section 10(d) are concerned with the col-
lision protection of automobiles. References (7, 13, 15 and 122 - 124)
describe various energy absorbing systems, References (20, 53, 60, 107
and 128) are related to defense purposes, and Reference (129) is concerned
with the earthquake resistance of buildings. Simple rigid-plastic
analyses could also be used for some of the design problems for buildings
subjected to severe dynamic loads which were discussed in Reference (130).

This field is a rapidly expanding one, and many of the individual
topics mentioned briefly herein warrant an entire review article to convey

adequately the current status.   Nevertheless, it is hoped that this
survey article, together with the earlier one published in this journal (1),
provides an entree into the literature extant, and gives some insight
into the advantages and disadvantages of various experimental. numerical
and theoretical approaches to the solution of dynamic structural problems
which involve material inelasticity.

Acknowledgements

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4. TITLE (and Subtitle) Recent Progress in the Dynamic Plastic Behavior of Structures		5. TYPE OF REPORT & PERIOD COVERED Interim
		6. PERFORMING ORG. REPORT NUMBER 78-1
7. AUTHOR(s) Norman Jones		8. CONTRACT OR GRANT NUMBER(s) N00014-76-C-0195 Task NR064-510
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Ocean Engineering Massachusetts Institute of Technology Cambridge, Massachusetts 02139		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Structural Mechanics Program O.N.R. Arlington, Virginia 22217		12. REPORT DATE January 1978
		13. NUMBER OF PAGES 59
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Unlimited Distribution

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Dynamic
Plastic
Structures
Review

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This article attempts to review the literature on the dynamic plastic response of structures published during the last three years, since the last survey by the author was published in the Shock and Vibration Digest(1). The review focuses largely on the behavior of simple structural components such as beams, plates and shells subjected to dynamic loads which produce extensive plastic flow of the material. In particular, recent work on the behavior of ideal fibre-reinforced

beams, higher modal response of beams, the influence of transverse shear and rotatory inertia, approximate methods of analysis, rapidly heated structures, fluid-structure interaction and dynamic plastic buckling are discussed in detail. These topics are followed by a discussion of a few recent numerical studies on the dynamic plastic response of structures, and a brief survey of some recent experimental and theoretical investigations into the collision protection of vehicles.

M.I.T. Department of Ocean Engineering
Report No. 78-1

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Behavior of Structures by N. Jones, January
1978, 59 pgs.

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